

Magnetic field instability in gas of massive electrons driven by the electroweak interaction and the anomalous magnetic moment

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Abstract

We show that a non-vanishing electric current of massive electrons can flow along the external magnetic field in the case when these electrons electroweakly interact with background matter and possess nonzero anomalous magnetic moments. The explicit value of this current is calculated on the basis of the exact solution of the Dirac equation in the external fields. We demonstrate that a magnetic field appears to be unstable if this current is accounted for. Then we consider a particular case of a degenerate electron gas, corresponding to a neutron star, and show that a seed magnetic field can be significantly amplified. Finally we discuss the application of our results for explain the electromagnetic radiation of highly magnetized compact stars.

The problem of the magnetic field instability is important, e.g., in the context of the existence of strong astrophysical magnetic fields [1]. Besides the magnetohydrodynamics mechanisms for the generation of astrophysical magnetic fields, recently the approaches based on the elementary particle physics were proposed. These approaches mainly rely on the chiral magnetic effect (CME) [2], which consists in the generation of the anomalous current of massless charged particles along the magnetic field $\mathbf{J}_5 = \alpha_{\text{em}}(\mu_{\text{R}} - \mu_{\text{L}}) \mathbf{B}/\pi$, where $\alpha_{\text{em}} \approx 1/137$ is the fine structure constant and $\mu_{\text{R,L}}$ are the chemical potentials of right and left chiral fermions. If \mathbf{J}_5 is accounted for in the Maxwell equations, the magnetic field appears to be unstable and can experience a significant enhancement. The

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applications of CME for the generation of astrophysical and cosmological magnetic fields are reviewed in Ref. [3].

However, the existence of CME in astrophysical media is questionable. As found in Refs. [4, 5], \mathbf{J}_5 can be non-vanishing only if the mass of charged particles, forming the current, is exactly equal to zero, i.e. the chiral symmetry is restored. For the case of electrons the restoration of the chiral symmetry is unlikely at reasonable densities which can be found in astrophysics [6]. The chiral symmetry can be unbroken in quark matter owing to the strong interaction effects [7]. The magnetic fields generation in quark matter, which can exist in some compact stars, was discussed in Refs. [8, 9]. Nevertheless this kind of situation looks quite exotic.

Therefore the issue of the existence of an electric current $\mathbf{J} \sim \mathbf{B}$ for massive particles, which can lead to the magnetic field instability, is quite important for the explanation of astrophysical magnetic fields. One of the example of such a current in electroweak matter was proposed in Ref. [10]. However, the model developed in Ref. [10] implies the inhomogeneity of background matter. This fact imposes the restriction on the scale of the magnetic field generated.

In the present Letter, we discuss another scenario for the magnetic field instability. It involves the consideration of the electroweak interaction of massive fermions with background matter along with nonzero anomalous magnetic moments of these fermions. Note that the electroweak interaction implies the generic parity violation which can provide the magnetic field instability. Recently, the interpretation of CME in terms of the effective magnetic moment was considered in Ref. [11].

This Letter is organized as follows. First, we discuss the Dirac equation for a massive electron with a nonzero anomalous magnetic moment, electroweakly interacting with background matter under the influence of an external magnetic field. Using the previously obtained solution of this Dirac equation, we calculate the electric current of these electrons along the magnetic field direction. This current turns out to be nonzero. Then we consider a particular situation of a strongly degenerate electron gas, which can be found inside a neutron star (NS). Finally we apply our results for the description of the amplification of the magnetic field in NS and briefly discuss the implication of our findings to explain the electromagnetic radiation of compact stars.

Let us consider an electron with the mass m and the anomalous magnetic moment μ . This electron is taken to interact electroweakly with nonmoving and unpolarized background matter consisting of neutrons and protons under the influence of the external magnetic field along the z -axis, $\mathbf{B} = B\mathbf{e}_z$. Accounting for the forward scattering off background fermions in the Fermi approximation, the Dirac equation for the electron has the form,

$$\{\gamma_\mu P^\mu - m - \mu B \Sigma_3 - \gamma^0 [V_R (1 + \gamma^5) + V_L (1 - \gamma^5)] / 2\} \psi = 0, \quad (1)$$

where $\gamma^\mu = (\gamma^0, \boldsymbol{\gamma})$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and $\Sigma_3 = \gamma^0\gamma^3\gamma^5$ are the Dirac matrices, $P^\mu = i\partial^\mu + eA^\mu$, $A^\mu = (0, 0, Bx, 0)$ is the vector potential, and $e > 0$ is the absolute value of the elementary charge. The effective potentials of the electroweak interaction $V_{R,L}$ have the

form [12],

$$\begin{aligned} V_R &= -\frac{G_F}{\sqrt{2}} [n_n - n_p(1 - 4\xi)] 2\xi, \\ V_L &= -\frac{G_F}{\sqrt{2}} [n_n - n_p(1 - 4\xi)] (2\xi - 1), \end{aligned} \quad (2)$$

where $n_{n,p}$ are the number densities of neutrons and protons, $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant, and $\xi = \sin^2 \theta_W \approx 0.23$ is the Weinberg parameter.

The solution of Eq. (1) has the form [13], $\psi = \exp(-iEt + ip_y y + ip_z z) (C_1 u_{n-1}, iC_2 u_n, C_3 u_{n-1}, iC_4 u_n)^T$, where $-\infty < p_{y,z} < +\infty$, $u_n(\eta) = (eB/\pi)^{1/4} \exp(-\eta^2/2) H_n(\eta)/\sqrt{2^n n!}$, with $n = 0, 1, \dots$, are the Hermite functions $H_n(\eta)$ are the Hermite polynomials, $\eta = \sqrt{eB}x + p_y/\sqrt{eB}$, and C_i , with $i = 1, \dots, 4$ are the spin coefficients. For the definiteness, here we use the chiral representation for the Dirac matrices. It is convenient to normalize the wave function ψ as

$$\int d^3x \psi_{p_y p_z n}^\dagger \psi_{p'_y p'_z n'} = \delta(p_y - p'_y) \delta(p_z - p'_z) \delta_{nn'}. \quad (3)$$

The energy levels E for $n > 0$ have the form [13],

$$\begin{aligned} E &= \bar{V} + \mathcal{E}, \quad \mathcal{E} = \sqrt{p_z^2 + m^2 + 2eBn + (\mu B)^2 + V_5^2 + 2sR^2}, \\ R^2 &= \sqrt{(p_z V_5 - \mu B m)^2 + 2eBn [V_5^2 + (\mu B)^2]}, \end{aligned} \quad (4)$$

where $s = \pm 1$ is the discrete spin quantum number, $\bar{V} = (V_L + V_R)/2$, and $V_5 = (V_L - V_R)/2$. At $n = 0$, the energy spectrum reads

$$E = \bar{V} + \sqrt{(p_z + V_5)^2 + (m - \mu B)^2}. \quad (5)$$

It should be noted that, at lowest energy level, the electron spin has only one direction since $C_1 = C_3 = 0$. In Eqs. (4) and (5) we take into account only the particle degrees of freedom.

Using the exact solution of the Dirac equation, we can calculate the electric current of electrons in this matter. This current has the form [4],

$$\mathbf{J} = -e \sum_{n=0}^{\infty} \sum_s \int_{-\infty}^{+\infty} dp_y dp_z \bar{\psi} \boldsymbol{\gamma} \psi f(E - \chi), \quad (6)$$

where $f(E) = [\exp(\beta E) + 1]^{-1}$ is the Fermi-Dirac distribution function, $\beta = 1/T$ is the reciprocal temperature, and χ is the chemical potential. First, we notice that $J_{x,y} \sim \bar{\psi} \gamma^{1,2} \psi = 0$ because of the orthogonality of the Hermite functions. The contribution of the lowest energy level with $n = 0$ to the electric current along the magnetic field $J_z \sim \bar{\psi} \gamma^3 \psi$

is vanishing: $J_z^{(n=0)} = 0$. This result is valid for arbitrary parameters m , μ , V_5 , and χ , as well as the magnetic field strength B .

The contributions of the higher energy levels with $n > 0$ to J_z can be obtained using the expressions for the spin coefficients C_i also found in Ref. [13],

$$J_z^{(n>0)} = -\frac{e^2 B}{(2\pi)^2} \sum_{n=1}^{\infty} \sum_{s=\pm 1} \int_{-\infty}^{+\infty} \frac{dp_z}{\mathcal{E}} \left[p_z \left(1 + s \frac{V_5^2}{R^2} \right) - s \frac{\mu B m V_5}{R^2} \right] f(E - \chi). \quad (7)$$

The first nonzero term in Eq. (7) is proportional to μB and V_5 ,

$$J_z = \mu m V_5 B^2 \frac{e^2}{\pi^2} \sum_{n=1}^{\infty} \int_0^{+\infty} \frac{dp}{\mathcal{E}_{\text{eff}}^2} \left[\left(1 - \frac{3p^2}{\mathcal{E}_{\text{eff}}^2} \right) \left(f' - \frac{f}{\mathcal{E}_{\text{eff}}} \right) + \frac{p^2}{\mathcal{E}_{\text{eff}}} f'' \right], \quad (8)$$

where $\mathcal{E}_{\text{eff}} = \sqrt{p^2 + m_{\text{eff}}^2}$ and $m_{\text{eff}} = \sqrt{m^2 + 2eBn}$. The argument of the distribution function in Eq. (8) is $\mathcal{E}_{\text{eff}} + \bar{V} - \chi$.

Let us consider the case of a strongly degenerate electron gas. In this situation $f = \theta(\chi - \bar{V} - \mathcal{E}_{\text{eff}})$, where $\theta(z)$ is the Heaviside step function. We can also disregard the positrons contribution to J_z . The direct calculation of the current in Eq. (8) gives

$$J_z = -2\mu m V_5 B^2 \frac{e^2}{\pi^2 \tilde{\chi}^3} \sum_{n=1}^{\infty} \sqrt{\tilde{\chi}^2 - m_{\text{eff}}^2} \theta(\tilde{\chi} - m_{\text{eff}}), \quad (9)$$

where $\tilde{\chi} = \chi - \bar{V}$. One can see that J_z in Eq. (9) is nonzero if $B < \tilde{B}$, where $\tilde{B} = (\tilde{\chi}^2 - m^2)/2e$. If the magnetic field is relatively strong and is close to \tilde{B} , then only the first energy level with $n = 1$ contributes to J_z , giving one $J_z = -8\mu m V_5 B^2 \alpha_{\text{em}} \sqrt{\tilde{\chi}^2 - m^2 - 2eB}/\pi \tilde{\chi}^3 \rightarrow 0$, where $\alpha_{\text{em}} = e^2/4\pi$. In the opposite situation, when $B \ll \tilde{B}$, one gets that $J_z = -8\alpha_{\text{em}} \mu m V_5 B (\tilde{\chi}^2 - m^2 - 2eB)^{3/2}/3\pi e \tilde{\chi}^3 \approx -8\alpha_{\text{em}} \mu m V_5 B/3\pi e$, i.e. the current is proportional to the magnetic field strength.

Returning to the vector notations we get that

$$\mathbf{J} = \Pi \mathbf{B}, \quad \Pi = -8\mu m V_5 B \frac{\alpha_{\text{em}}}{\pi \tilde{\chi}^3} \sum_{n=1}^N \sqrt{\tilde{\chi}^2 - m_{\text{eff}}^2}, \quad (10)$$

where N is maximal integer, for which $\tilde{\chi}^2 - m^2 - 2eBN \geq 0$. To study the magnetic field evolution in the presence of the additional current in Eq. (10) we take this current into account in the Maxwell equations along with the usual ohmic current $\mathbf{J} = \sigma_{\text{cond}} \mathbf{E}$, where σ_{cond} is the matter conductivity and \mathbf{E} is the electric field. Considering the magnetohydrodynamic approximation, which reads $\sigma_{\text{cond}} \gg \omega$, where ω is the typical frequency of the electromagnetic fields variation, we derive the modified Faraday equation for the magnetic field evolution,

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma_{\text{cond}}} \nabla \times (\Pi \mathbf{B}) + \frac{1}{\sigma_{\text{cond}}} \nabla^2 \mathbf{B}, \quad (11)$$

where we neglect the coordinate dependence of σ_{cond} .

Let us consider the evolution of the magnetic field given by the Chern-Simons wave, with the amplitude $A(t)$, corresponding to the maximal negative helicity, $\mathbf{A}(z, t) = A(t)(\mathbf{e}_x \cos kz + \mathbf{e}_y \sin kz + \mathbf{e}_z)$ or $\mathbf{B} = B(t)(\mathbf{e}_x \cos kz + \mathbf{e}_y \sin kz)$, where $k = 1/L$ is the wave number determining the length scale of the magnetic field L and $B(t) = -kA(t)$. In this situation we can neglect the coordinate dependence of Π in Eq. (11) and the equation for B takes the form,

$$\dot{B} = -\frac{k}{\sigma_{\text{cond}}}(k + \Pi)B. \quad (12)$$

Since Π in Eq. (10) is negative, the magnetic field described by Eq. (12) can be unstable.

We shall apply Eq. (10) to describe the magnetic field amplification in a dense degenerate matter which can be found in NS. In this situation, $n_n = 1.8 \times 10^{38} \text{ cm}^{-3}$ and $n_p \ll n_n$. Using Eq. (2) for this number density of neutrons, one gets that $V_5 = 6 \text{ eV}$. The number density of electrons can reach several percent of the nucleon density in NS. We shall take that $n_e = 9 \times 10^{36} \text{ cm}^{-3}$ which gives one $\chi = 125 \text{ MeV}$ [14]. Thus electrons are ultrarelativistic and we can take that $\tilde{\chi} \approx \chi$. We shall study the magnetic field evolution in NS in the time interval $t_0 < t < t_{\text{max}}$, where $t_0 \sim 10^2 \text{ yr}$ and $t_{\text{max}} \sim 10^6 \text{ yr}$. In this time interval, NS cools down from $T_0 \sim 10^8 \text{ K}$ mainly by the neutrino emission [15]. In this situation, the matter conductivity in Eq. (12) becomes time dependent $\sigma_{\text{cond}}(t) = \sigma_0(t/t_0)^{1/3}$ [14], where $\sigma_0 = 2.7 \times 10^5 \text{ GeV}$. Here we use the chosen electron density.

We shall discuss amplification of the seed magnetic field $B_0 = 10^{12} \text{ G}$. In such strong magnetic fields, the anomalous magnetic moment of an electron was found in Ref. [16] to depend on the magnetic field strength. We can approximate μ as

$$\mu = \frac{e}{2m} \frac{\alpha_{\text{em}}}{2\pi} \left(1 - \frac{B}{B_c}\right), \quad (13)$$

where $B_c = m^2/e = 4.4 \times 10^{13} \text{ G}$. Note that Eq. (13) accounts for the change of the sign of μ at $B \approx B_c$ predicted in Ref. [16].

The evolution of the magnetic field for the chosen initial conditions is shown in Fig. 1 for different length scales. One can see that, starting from $B_0 = 10^{12} \text{ G}$, the magnetic field reaches the saturated strength $B_{\text{sat}} \approx 1.3 \times 10^{13} \text{ G}$. Thus, both quenching factors in Eqs. (10) and (13) are important. One can see in Fig. 1 that a larger scale magnetic field grows slower. The further enhancement of the magnetic field strength compared to $L = 10^3 \text{ cm}$ corresponding to Fig. 1(b) is inexpedient since the growths time would significantly exceed 10^6 yr . At such evolution times, NS cools down by the photon emission from the stellar surface rather than by the neutrino emission [15].

The energy source, powering the magnetic field growth shown in Fig. 1, can be the kinetic energy of the stellar rotation. To describe the energy transmission from the rotational motion of matter to the magnetic field one should take into account the advection term $\nabla(\mathbf{v} \times \mathbf{B})$ in the right hand side of Eq. (11). Here \mathbf{v} is the matter velocity. Moreover one should assume the differential rotation of NS [17]. For this purpose we should take that NS is not in a superfluid state. This case is not excluded by the observational data [18]. We have estimated the spin down of NS with the radius $R \sim 10 \text{ km}$

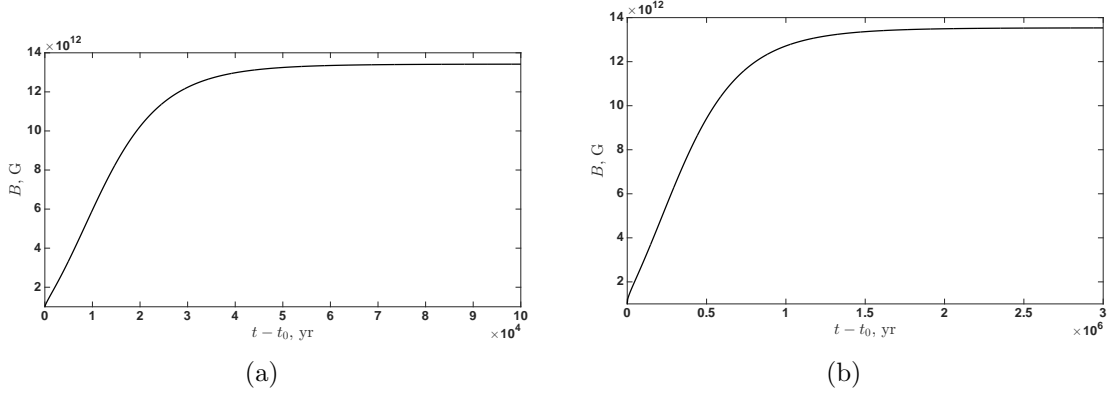


Figure 1: Magnetic field evolution obtained by the numerical solution of Eq. (12) for different length scales. (a) $L = 10^2$ cm, and (b) $L = 10^3$ cm.

and the initial rotation period $P_0 \sim 10^{-3}$ s basing on the conservation of the total energy $I\Omega^2/2 + B^2V/2 = \text{const}$, where I is the moment of inertia of NS, Ω is the angular velocity, and V is the NS volume. For $B_{\text{sat}} \approx 1.3 \times 10^{13}$ G shown in Fig. 1, the relative change of the period is $(P - P_0)/P_0 \sim 10^{-9}$.

The obtained results can be used for the explanation of electromagnetic flashes emitted by magnetars [19]. Magnetar bursts, happening in the stellar magnetosphere, are suggested in Ref. [20] to be triggered by plastic deformations of the magnetar crust driven by a thermoplastic wave. A thermoplastic wave can be excited by a small scale magnetic field fluctuation with $L \sim$ several meters [21] and $B \sim 10^{13}$ G [22]. As one can see in Fig. 1, these conditions are fulfilled in our case.

In conclusion we mention that, in the present Letter, we have considered the generation of the electric current of charged fermions, e.g., electrons, flowing along the external magnetic field. This current is nonzero if electrons electroweakly interact with background matter as well as if the nonzero mass and the nonzero anomalous magnetic moment are accounted for. Unlike the situation of massless fermions, when, owing to CME, $\mathbf{J}_5 \sim \mathbf{B}$ is created by the polarization effects at the lowest energy level [12, 14], in our case, only higher energy levels with $n > 0$ contribute to the current. We also mention that the role of a nonzero anomalous magnetic moment is crucial since, as found in Ref. [5], the current of massive charged particles electroweakly interacting with background matter is vanishing at any $n \geq 0$.

We have revealed that a magnetic field turns out to be unstable if the new current in Eq. (8) is taken into account. As an example of the obtained results, we have discussed the enhancement of the magnetic field in a dense degenerate matter. Using the background matter with characteristics typical to NS, we have obtained the amplification of the seed field $B_0 = 10^{12}$ G by more than one order of magnitude. The generated magnetic field is relatively small scale with $L \sim (10^2 - 10^3)$ cm. The time for the field growth is $(10^5 - 10^6)$ yr depending on the length scale. Finally we have considered the implication of our results for the explanation of magnetar bursts.

I am thankful to A. E. Lobanov for useful communications, as well as to the Tomsk State University Competitiveness Improvement Program and RFBR (research project No. 15-02-00293).

References

- [1] H. C. Spruit, The source of magnetic fields in (neutron-) stars, *Proc. Int. Astron. Union* **4**, 61 (2008).
- [2] V. A. Miransky and I. A. Shovkovy, Quantum field theory in a magnetic field: From quantum chromodynamics to graphene and Dirac semimetals, *Phys. Rept.* **576**, 1 (2015), arXiv:1503.00732.
- [3] D. E. Kharzeev, Topology, magnetic field, and strongly interacting matter, *Annu. Rev. Nucl. Part. Sci.* **65**, 193 (2015), arXiv:1501.01336.
- [4] A. Vilenkin, Equilibrium parity violating current in a magnetic field, *Phys. Rev. D* **22**, 3080 (1980).
- [5] M. Dvornikov, Role of particle masses in the magnetic field generation driven by the parity violating interaction, *Phys. Lett. B* **760**, 406 (2016), arXiv:1608.04940.
- [6] V. A. Rubakov, On the electroweak theory at high fermion density, *Prog. Theor. Phys.* **75**, 366 (1986).
- [7] M. Buballa and S. Carignano, Inhomogeneous chiral symmetry breaking in dense neutron-star matter, *Eur. Phys. J. A* **52**, 57 (2016), arXiv:1508.04361.
- [8] M. Dvornikov, Generation of strong magnetic fields in dense quark matter driven by the electroweak interaction of quarks, *Nucl. Phys. B* **913**, 79 (2016), arXiv:1608.04946.
- [9] M. Dvornikov, Magnetic fields in turbulent quark matter and magnetar bursts, arXiv:1612.06540.
- [10] V. B. Semikoz and D. D. Sokoloff, Large-scale magnetic field generation by α effect driven by collective neutrino-plasma interaction, *Phys. Rev. Lett.* **92**, 131301 (2004), astro-ph/0312567.
- [11] D. E. Kharzeev, M. A. Stephanov, H.-U. Yee, Anatomy of chiral magnetic effect in and out of equilibrium, *Phys. Rev. D* **95**, 051901 (2017), arXiv:1612.01674.
- [12] M. Dvornikov and V. B. Semikoz, Magnetic field instability in a neutron star driven by the electroweak electron-nucleon interaction versus the chiral magnetic effect, *Phys. Rev. D* **91**, 061301 (2015), arXiv:1410.6676.

- [13] I. A. Balantsev, A. I. Studenikin, and I. V. Tokarev, New solutions to the Dirac equation for particles in a magnetic field and a medium, *Phys. Part. Nucl.* **43**, 727 (2012).
- [14] M. Dvornikov and V. B. Semikoz, Generation of the magnetic helicity in a neutron star driven by the electroweak electron-nucleon interaction, *J. Cosmol. Astropart. Phys.* 05 (2015) 032, arXiv:1503.04162.
- [15] D. G. Yakovlev and C. J. Pethick, Neutron star cooling, *Ann. Rev. Astron. Astrophys.* **42**, 169 (2004), astro-ph/0402143.
- [16] I. M. Ternov, V. G. Bagrov, V. A. Bordovitsyn, and O. F. Dorofeev, Concerning the anomalous magnetic moment of the electron, *Sov. Phys. JETP* **28**, 1206 (1969).
- [17] S. L. Shapiro, Differential rotation in neutron stars: Magnetic braking and viscous damping, *Astrophys. J.* **544**, 397 (2000), astro-ph/0010493.
- [18] O. Y. Gnedin, D. G. Yakovlev, and A. Y. Potekhin, *Mon. Not. Roy. Astron. Soc.* **324**, 725 (2001), astro-ph/0012306.
- [19] R. Turolla, S. Zane, and A. L. Watts, Magnetars: the physics behind observations. A review, *Rep. Prog. Phys.* **78**, 116901 (2015), arXiv:1507.02924.
- [20] A. M. Beloborodov and Yu. Levin, Thermoplastic waves in magnetars, *Astrophys. J. Lett.* **794**, L24 (2014), arXiv:1406.4850.
- [21] X. Li, Yu. Levin, and A. M. Beloborodov, Magnetar outbursts from avalanches of Hall waves and crustal failures, *Astrophys. J.* **833**, 189 (2016), arXiv:1606.04895.
- [22] S. K. Lander, Magnetar field evolution and crustal plasticity, *Astrophys. J. Lett.* **824**, L21 (2016), arXiv:1604.02972.